OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4725

Further Pure Mathematics 1

Thursday **8 JUNE 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

 [Turn over

1 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$.

(i) Find $\mathbf{A} + 3\mathbf{B}$. [2]

(ii) Show that $\mathbf{A} - \mathbf{B} = k\mathbf{I}$, where $\mathbf{I}$ is the identity matrix and k is a constant whose value should be stated. [2]

2 The transformation S is a shear parallel to the x-axis in which the image of the point $(1, 1)$ is the point $(0, 1)$.

(i) Draw a diagram showing the image of the unit square under S. [2]

(ii) Write down the matrix that represents S. [2]

3 One root of the quadratic equation $x^2 + px + q = 0$, where p and q are real, is the complex number $2 - 3\mathrm{i}$.

(i) Write down the other root. [1]

(ii) Find the values of p and q. [4]

4 Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (r^3 + r^2) = \tfrac{1}{12}n(n+1)(n+2)(3n+1).$$ [5]

5 The complex numbers $3 - 2\mathrm{i}$ and $2 + \mathrm{i}$ are denoted by z and w respectively. Find, giving your answers in the form $x + \mathrm{i}y$ and showing clearly how you obtain these answers,

(i) $2z - 3w$, [2]

(ii) $(\mathrm{i}z)^2$, [3]

(iii) $\dfrac{z}{w}$. [3]

6 In an Argand diagram the loci C_1 and C_2 are given by

$$|z| = 2 \qquad \text{and} \qquad \arg z = \tfrac{1}{3}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence find, in the form $x + \mathrm{i}y$, the complex number representing the point of intersection of C_1 and C_2. [2]

7 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) Find $\mathbf{A}^2$ and $\mathbf{A}^3$. [3]

(ii) Hence suggest a suitable form for the matrix $\mathbf{A}^n$. [1]

(iii) Use induction to prove that your answer to part (ii) is correct. [4]

8 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} a & 4 & 2 \\ 1 & a & 0 \\ 1 & 2 & 1 \end{pmatrix}$.

(i) Find, in terms of a, the determinant of $\mathbf{M}$. [3]

(ii) Hence find the values of a for which $\mathbf{M}$ is singular. [3]

(iii) State, giving a brief reason in each case, whether the simultaneous equations

$$\begin{aligned} ax + 4y + 2z &= 3a, \\ x + ay &= 1, \\ x + 2y + z &= 3, \end{aligned}$$

have any solutions when

(a) $a = 3$,

(b) $a = 2$.

[4]

9 (i) Use the method of differences to show that

$$\sum_{r=1}^{n} \{(r+1)^3 - r^3\} = (n+1)^3 - 1. \qquad [2]$$

(ii) Show that $(r+1)^3 - r^3 \equiv 3r^2 + 3r + 1$. [2]

(iii) Use the results in parts (i) and (ii) and the standard result for $\sum_{r=1}^{n} r$ to show that

$$3\sum_{r=1}^{n} r^2 = \tfrac{1}{2}n(n+1)(2n+1). \qquad [6]$$

10 The cubic equation $x^3 - 2x^2 + 3x + 4 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

The cubic equation $x^3 + px^2 + 10x + q = 0$, where p and q are constants, has roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$.

(ii) Find the value of p. [3]

(iii) Find the value of q. [5]

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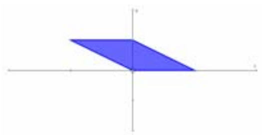
1.	i) $\begin{pmatrix} 7 & 4 \\ 0 & -1 \end{pmatrix}$	B1 B1	 2	Two elements correct All four elements correct
	(ii) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	B1		**A** – **B** correctly found
	$k = 3$	B1	2 **4**	Find k
2	(i) 	M1 A1	 2	For 2 other correct vertices For completely correct diagram
	(ii) $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$	B1 B1	2 **4**	Each column correct
3.	(i) 2 + 3i	B1	1	Conjugate seen
	(ii)	M1		Attempt to sum roots or consider x terms in expansion or substitute 2 – 3i into equation and equate imaginary parts
	p = -4	A1		Correct answer
		M1		Attempt at product of roots or consider last term in expansion or consider real parts
	q = 13	A1	4 **5**	Correct answer

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4.	$\Sigma r^3 + \Sigma r^2$	M1		Consider the sum as two separate parts
	$\Sigma r^2 = \frac{1}{6}n(n+1)(2n+1)$	A1		Correct formula stated
	$\Sigma r^3 = \frac{1}{4}n^2(n+1)^2$	A1		Correct formula stated
	$\frac{1}{12}n(n+1)(n+2)(3n+1)$	M1		Attempt to factorise and simplify or expand both expressions
		A1	5	Obtain given answer correctly or complete verification
			5	
5.	(i) -7i	B1		Real part correct
		B1	2	Imaginary part correct
	(ii) 2 + 3i	B1		iz stated or implied or $i^2 = -1$ seen
		B1		Real part correct
	-5 + 12i	B1	3	Imaginary part correct
	(iii) $\frac{1}{5}(4 - 7\text{i})$ or equivalent	M1		Multiply by conjugate
		A1		Real part correct
		A1	3	Imaginary part correct
			8	**N.B. Working must be shown**
6..	(i) Circle , Centre *O* radius 2	B1 B1		Sketch showing correct features
	One straight line	B1		
	Through *O* with +ve slope	B1		
	In 1st quadrant only	B1	5	
	(ii) $1 + i\sqrt{3}$	M1		Attempt to find intersections by trig, solving equations or from graph

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		A1	2 **7**	Correct answer stated as complex number
7.	(i) $\mathbf{A}^2 = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$ $\mathbf{A}^3 = \begin{pmatrix} 8 & 0 \\ 0 & 1 \end{pmatrix}$	M1 A1 A1	 3	Attempt at matrix multiplication Correct $\mathbf{A}^2$ Correct $\mathbf{A}^3$
	(ii) $\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 0 & 1 \end{pmatrix}$	B1	1	Sensible conjecture made
	(iii)	B1 M1 A1 A1	 4 **8**	State that conjecture is true for $n = 1$ or 2 Attempt to multiply $\mathbf{A}^n$ and $\mathbf{A}$ or vice versa Obtain correct matrix Statement of induction conclusion
8.	(i) $a\begin{bmatrix} a & 0 \\ 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & a \\ 1 & 2 \end{bmatrix}$ $a^2 - 2a$	M1 A1 A1	 3	Correct expansion process shown Obtain correct unsimplified expression Obtain correct answer
	(ii) $a = 0$ or $a = 2$	M1 A1A1ft	 3	Solve their det $\mathbf{M} = 0$ Obtain correct answers
	(iii) (a)	B1 B1		Solution, as inverse matrix exists or $\mathbf{M}$ non-singular or det$\mathbf{M} \neq 0$
	(b)	B1 B1	4 **10**	Solutions, eqn. 1 is multiple of eqn 3

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9.	(i)	M1 A1	 2	Show that terms cancel in pairs Obtain given answer correctly
	(ii)	M1 A1	 2	Attempt to expand and simplify Obtain given answer correctly
	(iii) $(n+1)^3 - 1 - \frac{3}{2}n(n+1) - n$	B1 B1 M1 M1 A1		Correct Σr stated $\Sigma 1 = n$ Consider sum of three separate terms on RHS Required sum is LHS – two terms Correct unsimplified expression
	$\frac{1}{2}n(n+1)(2n+1)$	A1	6 **10**	Obtain given answer correctly
10.	(i) $\alpha + \beta + \gamma = 2$ $\alpha\beta\gamma = -4$	B1 B1		Write down correct values
	$\alpha\beta + \beta\gamma + \gamma\alpha = 3$	B1	3	
	(ii)	M1		Sum new roots
	$\alpha + 1 + \beta + 1 + \gamma + 1 = 5$	A1ft		Obtain numeric value using their (i)
	p = -5	A1ft	3	p is negative of their answer
	(iii)	M1*		Expand three brackets
		A1		$\alpha\beta\gamma + \alpha\beta + \beta\gamma + \gamma\alpha + \alpha + \beta + \gamma + 1$
		DM1		Use their (i) results
		A1ft		Obtain 2
	q = -2	A1ft	5	q is negative of their answer
			11	**Alternative for (ii) & (iii)**
		M2 A1 M1 A2 A1 A1		Substitute $x = u - 1$ in given equation Obtain correct unsimplified equation for u Expand Obtain $u^3 - 5u^2 + 10u - 2 = 0$ State correct values of p and q.

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